Kuwait University | Math 102 | Date: October 23, 1997
Dept of Math and Comp Sci | First Midterm | Duration: 75 minutes

Answer all of the following questions. Calculators, pagers and mobile telephones are NOT allowed. Each (sub)question is worth 4 points.

1. Show that the inverse of $f(x) = \dfrac{\cos(e^x)+1}{2}$ $-\infty < x < \ln \pi$ exists, and find $f^{-1}(x)$ State the domain and range of f^{-1}.

2. Let $f(x) = \ln(\cos(x^3)) - 2x + 1$, $0 \le x \le 1$. Find an equation of the tangent line to the graph of f^{-1} at the point $(1, 0)$.

3. Write $\sin(\tan^{-1} x) + \tan(\sin^{-1} x)$ as an algebraic expression in x for $x > 0$

4. Show that $\dfrac{d}{dx}\sec^{-1} x = \dfrac{1}{x\sqrt{x^2-1}}$

5. Find $\dfrac{dy}{dx}$ if

a. $y = 3^{\sec^{-1} x}(\cosh x)^x$

b. $\sinh(\sec y) + \ln(e^x + 1) = \cosh y + \sin^{-1} x^2$

6. Evaluate the following integrals:

a. $\displaystyle\int \frac{\cos x \sin x}{\cos^2 x - 3}\,dx$

b. $\displaystyle\int \frac{\cos x}{1+\sin^2 x}\,dx$

c. $\displaystyle\int \frac{\ln(x^2 e^x)}{x}\,dx$

d. $\displaystyle\int \frac{dx}{x\sqrt{x^8-9}}$